Excitation of fast and slow magnetosonic waves by kinetic Alfven waves

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Abstract. The nonlinear parametric interaction of Alfven waves with magnetosonic and ion-acoustic waves is considered on the basis of two-fluid magnetohydrodynamics. A nonlinear dispersion relation describing three-wave interaction, instability growth rate have been calculated and estimated. The analyses of theoretical results shows that kinetic effects in the Alfven waves (the finite ion Larmour radius) are essential for the parametric interactions of waves.

Nonlinear parametric processes studied in the paper could take place in the solar coronal loops, where plasma parameter is small. The products of the decay - magnetosonic and ion-acoustic waves, can effectively heat the coronal plasma in consequence of rapid dissipation.

I INTRODUCTION

Magnetohydrodynamic waves play an important role in the dynamics of Earth magnetosphere, solar wind and atmosphere of the Sun. The long-period geomagnetic pulsations observed in the Earth’s magnetosphere have been considered as MHD Alfven waves ([1], [2]). The satellite observations show that the most part of turbulent pulsations are Alfven and magnetosonic waves in the solar wind [3]. They are thought to play an essential role in the solar wind heating. The MHD waves can provide a source of energy for corona heating. Alfven waves have enough energy for heating of coronal loops (see e.g. [4] and references therein). But they are slow-damped. This is the reson why Alfven wave can’t heat coronal loops directly, and the problem arise: how the energy can be transferred from waves to plasma particles. The ion-acoustic and magnetosonic waves are rapid damped waves. Therefore, if the propagating Alfven wave decays into ion-acoustic and magnetosonic waves, they can transfer rapidly their energy to particles in consequence of dissipation. In the paper [5] was shown that processes of formation of acoustic waves from Alfven waves are effective in the region, where \( V_A/V_s \approx 1/30 \). The dissipation of wave energy arises on distances which are comparable with length of coronal
loops. Nonlinear interaction between Alfven and ion-acoustic waves was considered for the case of linear dispersion, i.e., without accounting of thermal effect (finite Larmor radius of proton). But in the magnetosheeric and coronal plasma thermal effects play an important role. When the thermal effects are taken into account it leads to nonlinear dependence of frequency from wavenumber, which is essential for nonlinear wave interaction.

In the present work taking into account the thermal effect we consider nonlinear parametric interaction of Alfven wave with the ion-acoustic and magnetoionic waves.

II BASIC EQUATION

It is assumed that Alfven wave with finite amplitude propagates in homogeneous magnetized plasma ($\vec{B}_0 = B_0 \hat{e}_z$):

$$\vec{E}_0 = (E_{0x} \hat{e}_x + E_{0z} \hat{e}_z) \exp\left(-i(\omega_0 t + \vec{k}_0 \vec{r})\right) + c.c.,$$

where the frequency and wave vector are coupled by equality:

$$\omega_0^2 = k_{0z}^2 V_d^2 (1 + t \mu_i),$$

Here $\mu_i = k_{0z}^2 \rho_i^2$, $\rho_i = V_{Ti}/\omega_{Bi}$ - proton Larmor radius, $\omega_{Bi} = eB/m_i c$ - ion cyclotron frequency, $t = T_e/T_i$. Our governing equations are a set of two-fluid MHD equation:

$$\frac{\partial \vec{v}_e}{\partial t} = \frac{1}{m_e} (e_{e} \vec{E} + F_{e}) + (\vec{v}_e \times \vec{B}_{ei}) - \frac{T_{e}}{m_e n_e} \nabla n_e,$$

$$\frac{\partial n_e}{\partial t} = -\nabla (n_e \vec{v}_e),$$

$$\nabla \times \vec{B} = \frac{4\pi}{c} j + \frac{1}{c} \frac{\partial \vec{E}}{\partial t},$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t},$$

$$\nabla \cdot \vec{E} = 4\pi \rho,$$

where

$$\vec{j} = e(n_i \vec{v}_i - n_e \vec{v}_e),$$

$$\rho = e(n_i - n_e).$$
\[ \mathbf{F}_{\alpha} = \frac{e}{c} (\mathbf{v}_{\alpha} \times \mathbf{B}) - m_{\alpha} (\nabla \times \mathbf{v}_{\alpha}). \]

Index \( \alpha = i, e \) corresponds to the ion and electron components of plasma respectively. As \( F_{pi} = \frac{m_i}{m_e} F_{pe} \), influence of \( F_{pi} \) force is small, and can be neglected. In Ampere-Maxwell’s equation we also neglect the displacement current.

We consider equations in Cartesian coordinates \((x, y, z)\), supposing that all wave vectors are situated in OXZ plane. It is assumed that the wave synchronism conditions are satisfied:

\[ \omega_0 = \omega + \omega_1, \quad \mathbf{k}_0 = \mathbf{k} + \mathbf{k}_1, \]

where \( \omega_1, \mathbf{k}_1, \omega, \mathbf{k} \) - frequency and wave vector of magnetosonic and ion-acoustic wave respectively.

### III DISPERSION EQUATION FOR LOW-FREQUENCY PERTURBATIONS.

Since Alfvén and ion-acoustic waves are ultralow-frequency waves, we can use the plasma approximation for obtaining of a dispersion equation:

\[ \tilde{n}_e = \tilde{n}_i \]

where \( \tilde{n}_e \) and \( \tilde{n}_i \) are perturbations of ion and electron number densities. From the equation of motion and continuity equation, we can find expressions for \( \tilde{n}_e \) and \( \tilde{n}_i \):

\[ \frac{\dot{\tilde{n}}_e}{n_0} = -e \left( \psi - \frac{k_z^2 \omega^2}{k_z^2 \omega_{Be}^2} \phi - Q_{NL} \right), \]

\[ \frac{\dot{\tilde{n}}_i}{n_0} = -e_m \left( \frac{k_z^2 \psi_i}{\omega^2} - \frac{k_z^2 \phi_i}{\omega_{Pi}^2} \right) \]

Here

\[ Q_{NL} = \frac{k_z \omega}{ek_z^2 \omega_{Be}} \left( \frac{\omega}{\omega_{Be}} - F_y + F_y \right) + \frac{F_z}{ie k_z}. \]

To find the relation between \( \varphi \) and \( \psi \), we use z component of the Ampère’s law in two potential approximation and equation of charge conservation. Compare (10) to (11) and use relation between \( \varphi \) and \( \psi \), we obtain dispersion equation for ultralow-frequency perturbations:

\[ \left[ \frac{k_z^2 V_z^2}{\omega^2} \left( 1 - \frac{\omega^2}{k_z^2 V_A^2} \right) \left( 1 - \frac{\omega^2}{k_z^2 V_e^2} \right) - \mu_s \right] \varphi = -Q_{NL} \]

In the absence of pump wave \( (Q_{NL} = 0) \) we have:
\[ \omega_A^2 = k_A^2 V_A^2 (1 + \mu_s), \]
\[ \omega_s^2 = k_s^2 V_s^2 (1 + \mu_s). \]  

These expressions describe dispersion laws for Alfven and ion-acoustic waves respectively. The special properties of these waves are their ability to propagate in the \( xz \) plane, thus the wave energy can be transferred across the external magnetic field as well as along it. And since \( \omega_s^2 \ll \omega_A^2 \) and \( \omega_A^2 \simeq k_A^2 V_A^2 \), dispersion equation for ion-acoustic wave can be rewritten as:

\[ \varepsilon \phi = \eta \psi_0 E_{1y}^* \]  

where

\[ \varepsilon = \omega^2 - k_s^2 V_s^2 (1 + \mu_s), \]
\[ \eta = -\frac{\omega^2 k_0 e V_{1y}^*}{\omega_{Be} k_s V_A T_e}. \]

In the calculation of the coupling coefficient \( \eta \), we have taken account of the pondermotive force created by interaction of the pump wave with the magnetosonic wave.

### IV DISPERSION EQUATION FOR MAGNETOSONIC WAVE

In order to obtain a dispersion equation for magnetosonic wave, which propagate across magnetic field, we also use plasma approximation. In this case ions and electrons velocity must coincide. From the electron equation of motion we find:

\[ V_{ex} = \frac{e}{m_e \omega_{Be}} \left( E_{1y} + i \frac{\omega_i}{\omega_{Be}} E_{1x} \right) + \frac{1}{m_e \omega_{Be}} \left( i \frac{\omega_i}{\omega_{Be}} F_{1x} + F_{1y} \right), \]  

where term proportional to \( \omega / \omega_{Be} \) describes the inertial drift, that must be taken into account for wave that propagates across magnetic field. The component of pondermotive force is determined by interaction of pump wave and ion-acoustic wave. From the ion equation of motion and Maxwell's equation we find expression for \( V_{ix} \):

\[ V_{ix} = \frac{k_i^2 V_i^2}{\omega_i^2} \frac{e E_{1y}}{m_i \omega_{Be}}. \]  

Compare (15) to (16) we obtain dispersion equation for magnetosonic wave:

\[ \varepsilon_1 E_{1y} = \eta \psi \phi_0, \]  

where

\[ \varepsilon_1 = \omega_i^2 V_i^2 \]
and coupling coefficient is determined by

$$
\eta_1 = -2V_A V_S \frac{m_e e}{c} \left( \frac{V_T}{V_A} \right)^2 \frac{k_0 \omega_A}{\omega_B} \omega_0^2 \mu_c,
$$

$$
\mu_c = k_e^2 \rho_c^2.
$$

V NONLINEAR DISPERSION EQUATION AND DECAY GROWTH RATE

From ion-acoustic (14) and magnetosonic (17) dispersion equations we can find a nonlinear dispersion relation describing three-wave interaction:

$$
\varepsilon \varepsilon_1^* = \eta \eta_1^* | \phi_0 |^2.
$$

In the absence of the pump wave, $\phi_0 = 0$, two kinds of waves with frequencies $\omega = \omega_k$ and $\omega_1 = \omega_{jk}$ exist in the plasma. In the presence of the pump wave, $\phi_0 \neq 0$, energy transfer from the pump KAW to the other plasma modes occurs, and amplitudes increase with the growth rate. Putting $\omega = \omega_r + i\gamma$, $\omega_1 = \omega_{1r} + i\gamma$ ($|\gamma| \ll |\omega_r, \omega_{1r}|$) into (19) and expanding $\varepsilon$ and $\varepsilon_1$ in Taylor series, we obtain an expression for the wave growth rate:

$$
\gamma^2 = \frac{\eta \eta_1 | \phi_0 |^2}{\frac{\partial \varepsilon}{\partial \omega} \frac{\partial \varepsilon_1}{\partial \omega_1}} \bigg|_{\omega=\omega_r, \omega_1=\omega_{1r}},
$$

Where $\omega_r$ and $\omega_{1r}$ are defined from equations:

$$
\varepsilon(\omega_r, \vec{k}) = 0, \quad \varepsilon_1(\omega_1, \vec{k}_1) = 0.
$$

substituting the expressions for the coupling coefficients $\eta, \eta_1$ and derivative $\frac{\partial \varepsilon}{\partial \omega}, \frac{\partial \varepsilon_1}{\partial \omega_1}$ we find:

$$
\gamma^2 = \frac{W V_S}{2 \frac{\omega_0^2}{V_A} \omega_B k_z V_A} \left( \frac{\omega}{\omega_B} \right) \mu_c \omega_{1c}^2
$$

where

$$
W = \frac{|E_{0z}|}{4 \pi n_0 T_e}
$$
VI CONCLUSIONS

Let us estimate the instability growth rate for the coronal plasma. Spectroscopic observations suggest the presence of Alfvén waves with amplitudes $B/B_0 = 0.01$ in coronal loops [6]. To examine existence of Alfvén waves, these authors used the method first proposed by [7]. Typical parameters for such loop are length $L = (2 - 5) \times 10^9$ cm, $n = (0.5 - 1) \times 10^{10}$ cm$^{-3}$, $B = 100 - 500G$, $T = 4 \times 10^6 K$, $\omega_{pe}/\omega_{Be} \approx 10$, $\mu_e \approx 1$, $\omega_1 \approx 10^{-1}\omega_{Be}$. Substituting these values of the plasma parameters and a KAW intensity of $W \approx 10^{-5} - 10^{-6}$ into (21), we obtained $\gamma \approx (10^2 \div 10^3) c^{-1}$ and the time of instability development respectively $\tau = \gamma^{-1} \approx 0.01$ c. The nonlinear interaction of Alfvén waves with magnetosonic and ion-acoustic waves presented in this paper could take place in the Earth’s magnetosphere as well as in the solar magnetosphere, where a value of the plasma parameter $\beta$ is low. In solar atmosphere the sources of Alfvén waves can be 5-min. chromospheric oscillation, that swing the basis of magnetic loops and lead to propagation of alfvén waves into corona. Alfvén waves will transform into kinetic Alfvén waves during the propagation along magnetic field. The kinetic Alfvén wave can interact more effective with other kinds of waves because of the presence of a non-zero longitudinal component of the electric field $E_z$. When KAW reaches the region with low plasma parameter $\beta$ it will decay into magnetosonic and ion-acoustic waves. Due to their rapid dissipation, these waves are more effective for heating of the coronal plasma. The exitation of fast magnetosonic waves by phase mixing of Alfvén wave was considered in [8]. The relative efficacy exitation of slow and fast magnetosonic waves by Alfvén waves need further investigation.

REFERENCES