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Oblique Bernstein Mode Generation Near the Upper-hybrid Frequency in Solar Pre-flare Plasmas

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Abstract We study analytically the generation process of the first harmonics of the pure electron weakly oblique Bernstein modes. This mode can appear as a result of the rise and development of a corresponding instability in a solar active region. We assume that this wave mode is modified by the influence of pair Coulomb collisions and a weak large-scale sub-Dreicer electric field in the pre-flare chromosphere near the footpoints of a flare loop. To describe the pre-flare plasma we used the model of the solar atmosphere developed by Fontenla, Avrett, and Loeser (*Astrophys. J.* **406**, 319, 1993). We show that the generated first harmonic is close to the upper-hybrid frequency. This generation process begins at the very low threshold values of the sub-Dreicer electric field and well before the beginning of the preheating phase of a flare. We investigate the necessary conditions for the existence of non-damped first harmonics of oblique Bernstein waves with small amplitudes in the flare area.

Keywords Pre-flare chromosphere · Plasma instabilities · Microwave emission

1. Introduction

The understanding of the physical connection between the solar flare process and microwave emission from an active region (AR) has been an interesting and important problem of modern solar physics for several decades. Based on the electro-magnetic (EM) radiation in the frequency range 1 – 100 GHz generated at the footpoint of a magnetic loop, we consider the possibility of predicting in a short time interval the flare appearance in an AR. We analyse

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the potential mechanisms of plasma turbulence generation through plasma instabilities that appear before solar flare eruptions. We have made a few important assumptions. First, we assume a weak and quasi-static electric field in the magnetic loop. The existence of such a field has been confirmed by the additional Stark broadening of Balmer H_β lines with principal quantum numbers $N > 8$ (see *e.g.* Foukal and Hinata, 1991). Furthermore, we also assume that energetic particle beams are not present in the lower and middle chromosphere, as this allows us to investigate the pre-flare stage in regions where the first energy release is not observed in general (see *e.g.* Hudson, 2007; Melnikov, Shibasaki, and Reznikova, 2002). We note that the findings presented here can be directly compared with previous observational results in a limited number of cases (see *e.g.* Bogod and Yasnov, 2009; Zharkova *et al.*, 2011; Kashapova, Meshalkina, and Kisil, 2012). In contrast, the majority of studies of microwave radiation from ARs consider either impulsive or gradual processes: flare or post-flare conditions with beams of energetic particles and without large-scale, quasi-static, sub-Dreicer (see *e.g.* Miller *et al.*, 1997) electric fields.

To explain a burst of microwave emission from solar magnetic loop structures, Holman, Eichler, and Kundu (1980) proposed the gyrosynchrotron masering mechanism. The authors pointed out that the observed GHz frequency of the gyrosynchrotron emission requires a magnetic field with a strength of a few kilogauss. Vlahos (1987) mentioned, among other characteristics of electron-cyclotron maser emission (ECME), that the frequency of emitted radiation is present around the first or second harmonics of the cyclotron frequency. For solar atmospheric parameters, the frequency of emitted radiation is equal to $2-3$ GHz. The properties of the ECME at different harmonics of the electron-cyclotron frequency, Ω_e, in the range $\omega = 1.16-1.69$ GHz have been studied by Krucker and Benz (1994). Between 0.3 and 3 GHz, the microwave emission has been detected at frequencies, which are close enough, but do not coincide with integer values of Ω_e. A possible explanation of this has been proposed by Willes and Robinson (1996). These authors assumed that several adjacent Bernstein modes can be excited by the cyclotron maser mechanism and radiation can escape from the solar atmosphere as a result of non-linear wave–wave interactions. Bogod *et al.* (2000) suggested the presence of emission in a close-frequency range, such as thermal cyclotron radiation at the third harmonics of Ω_e from a compact source in an solar AR. This mechanism can be considered as an alternative to ECME. Melnikov, Shibasaki, and Reznikova (2002) have analysed the microwave sources in extended solar flaring loops at 17 and 34 GHz. The authors studied the origin of enhanced losses of trapped particles in the lower part of a magnetic loop and proposed that this process can be explained by two possible mechanisms: (i) Coulomb collisions in a growing plasma density or (ii) scattering on locally (near footpoint) enhanced wave turbulence. This confirms that the overwhelming majority of the observational flare data in magnetic loops correspond to the impulsive and gradual phases of this process. Exceptions are very rare, but exist. For example, Bogod and Yasnov (see *e.g.* 2009) reported on spikes of microwave emission from NOAA AR 9077 in its pre-flare stage during four days before the Bastille Day flare, which occurred on 14 July 2000. The observations had been carried out in the frequency band from 4 to 16 GHz. Calculations and estimates have shown that the radiating region was probably surrounded by a magnetic field with $|\mathbf{B}_0| < 10-40$ G (see *e.g.* Bogod and Yasnov, 2009). Kryshtal, Gerasimenko, and Voitsekhovska (2012) pointed out that values of the electron-cyclotron frequency in the range $1-40$ GHz correspond to magnetic fields in the kilogauss range (in S. Solanki's terminology, see *e.g.* Solanki, 1993). Hudson (2007) proposed using 10 G and 1000 G as representative values for the magnetic field strength of chromospheric flares. In this article, we have considered magnetic field strengths of the order of kilogauss (see *e.g.* Solanki, 1993). The presence of a sub-Dreicer field at the early stage of the flaring process

(*i.e.* pre-flare phase) can initiate a number of plasma wave instabilities (see *e.g.* Kryshtal and Gerasimenko, 2004; Kryshtal, Gerasimenko, and Voitsekhovska, 2012; Kryshtal *et al.*, 2013; Kryshtal, Gerasimenko, and Voitsekhovska, 2014). The instabilities with extremely low threshold of excitation are most important (Kryshtal, Gerasimenko, and Voitsekhovska, 2012). We note that electromagnetic-wave emission during the linear stage of the instability development, assuming a small growth rate, can be generated by nonlinear parametric decay or wave coalescence.

2. Wave Sources and Three-wave Interactions

To explain the presence of non-integer ratio emission frequencies in solar microwave spike bursts, Willes and Robinson (1996) have proposed to take into account the possibility of the generation of Bernstein modes. These adjacent modes can be excited through the cyclotron-maser mechanism, but the question is how their generation can be observed in remote measurements. The authors proposed non-linear wave–wave interactions as a solution of this problem. The coalescence of two Bernstein modes (B_1 and B_2) may result in the appearance of an electromagnetic wave (EMW), *i.e.,*

$$B_1 + B_2 \rightarrow \text{EMW}, \tag{1}$$

which can be remotely measured. Willes and Robinson (1996) have considered the electron loss-cone-like distribution function. The authors have shown that the process represented by formula (1) is more probable when B_1 and B_2 are the same Bernstein modes. Kryshtal (1998) has proposed another mechanism of generation of second Bernstein harmonics as a result of an instability in the pre-flare plasma. This author assumed that the weak sub-Dreicer electric field in the loop footpoint can be the main driver of the Bernstein wave instability. We note that Kryshtal (1998) did not consider a pure Bernstein mode, but an oblique one. Therefore, Landau damping together with pair Coulomb collisions could suppress the instability. Furthermore, this process was investigated by Kryshtal, Gerasimenko, and Voitsekhovska (2012). These authors have found that the excitation threshold of this instability is low with respect to the amplitude of the sub-Dreicer electric field. For the three-wave interactions, the process of wave coalescence is less probable than the decay processes (see *e.g.* Alexandrov, Bogdankevich, and Rukhadze, 1984). Yukhimuk *et al.* (1998) have shown that the parametric decay of the upper-hybrid pump wave (UHW) into a kinetic Alfvén wave (KAW) and an EMW is possible in the solar atmosphere, *i.e.*,

$$\text{UHW} \rightarrow \text{KAW} + \text{EMW}. \tag{2}$$

The generation of the first harmonic of the oblique Bernstein modes mentioned above, due to the rise and development of a corresponding instability, has been demonstrated in the article by Kryshtal (1997). For realistic values of the main physical parameters of the pre-flare plasma this harmonic becomes very close to the upper-hybrid mode. These two waves can be described by practically the same dispersion relation (see *e.g.* Alexandrov, Bogdankevich, and Rukhadze, 1984; Krall and Trivelpiece, 1973). Under additional conditions (see *e.g.* Chen, 1984), the generated EMW could be leaky, *i.e.* the generated waves can leave the generation area and, as a result, can be detected. Therefore, the identification of the necessary conditions for the development of low-threshold instabilities becomes an important problem for short-term flare prediction. There are two different types of short-term flare forecast in ARs. Conventionally, we can call them synoptic and causal forecast. The first approach is well developed and could be applied for most cases. The synoptic approach is

based on evidence observed in the AR before the flare, such as (i) the maximum horizontal gradient of the magnetic field, (ii) the length of the neutral line as an indicator of flare productivity, and (iii) the number of singular points (see *e.g.* Yu *et al.*, 2010; Li and Zhu, 2013). The causal forecast is based on a model of the flaring process (see *e.g.* Willes and Robinson, 1996; Miller *et al.*, 1997; Bogod *et al.*, 2000; Kryshtal, Gerasimenko, and Voitsekhovska, 2012). The most realistic forecast should include both approaches (see *e.g.* Núñez *et al.*, 2005). However, this requires more accurate high-resolution measurements. The instability parameters, theoretically obtained in this article, and the observational results form other authors can only be directly compared when the microwave radiation has been registered during the pre-flare stage in the AR (see *e.g.* Bogod and Yasnov, 2009; Zharkova *et al.*, 2011; Kashapova, Meshalkina, and Kisil, 2012).

In comparison with the generation of second harmonics (see *e.g.* Kryshtal, 1998; Kryshtal, Gerasimenko, and Voitsekhovska, 2012), the excitation of the first harmonic is more likely because of its higher probability. To confirm this, it is necessary to obtain the instability threshold value in the vicinity of upper-hybrid frequency and to show that this value is extremely low. In this article, we obtain the boundary values of the main characteristics of pre-flare plasma and wave perturbation, and investigate the possibility of the generation of non-damped oscillations (waves) with small amplitudes. The comparison of the excitation threshold values for the first and second harmonics is also an important problem. For example, the beginning of EM radiation at the electron-cyclotron harmonics with non-integer numbers (Willes and Robinson, 1996) is possible if the threshold values of the first harmonic are of the same order as the second. On the other hand, when these threshold values are significantly different, the harmonic with a lower threshold value will be generated earlier with no influence of another harmonic.

In this article, we propose the necessary conditions for a causal short-term forecast, *i.e.* conditions for emergence of microwave radiation in the solar ARs before a flare. We investigate solely the linear stage of the perturbation development since it is the most important and relevant for forecasting.

3. Equations and Approximations

We refer to the plasma as pre-flare plasma if the electron stream velocity $\mathbf{u}$ (sometimes called drift velocity) with respect to the almost immovable ions is much lower than the electron thermal velocity v_{Te}. In a plasma with a quasi-static external electric field, $\mathbf{E}_0$, for a time interval when the electron stream is effectively decelerated by the ions and, therefore, the percentage of runaway electrons is negligibly small, $\mathbf{u}$ can be expressed as (see *e.g.* Alexandrov, Bogdankevich, and Rukhadze, 1984)

$$\mathbf{u} = \frac{e\mathbf{E}_0}{m_{\mathrm{e}} v_{\mathrm{ei}}}, \tag{3}$$

where $|\mathbf{E}_0|$ is the amplitude of electric field, m_{e} and e are the electron mass and charge, and v_{ei} is the frequency of electron–ion collisions. For pair Coulomb collisions, this frequency has the form (see *e.g.* Chen, 1984)

$$v_{\mathrm{ei}} = 2 \times 10^{-6} n \frac{\ln \Lambda}{T_{\mathrm{e,eV}}^{3/2}}. \tag{4}$$

In this equation, n is the plasma particle number density, $\ln \Lambda$ is the Coulomb logarithm, and $T_{\mathrm{e,eV}}$ is the (kinetic) electron temperature expressed in electron-volts. By applying the

well-known formula for the amplitude of the local Dreicer field (see *e.g.* Alexandrov, Bogdankevich, and Rukhadze, 1984),

$$E_{\mathrm{D}} = \frac{m_{\mathrm{e}}}{e} V_{\mathrm{Te}} \nu_{\mathrm{ei}},$$

the plasma can be assumed to be a pre-flare plasma if it satisfies the following condition:

$$\varepsilon_{\mathrm{R}} \equiv \frac{|E_0|}{E_{\mathrm{D}}} = \frac{u}{V_{\mathrm{Te}}} \ll 1. \tag{5}$$

If Equation (5) is satisfied, the field is called sub-Dreicer field (see *e.g.* Miller *et al.*, 1997). According to the flare model proposed by Heyvaerts, Priest, and Rust (1977), the earliest (*i.e.* pre-heating) stage of a flare process precedes the flash (or impulsive) phase. During this pre-heating phase the Buneman plasma instability (see *e.g.* Alexandrov, Bogdankevich, and Rukhadze, 1984; Chen, 1984) develops if the stream velocity **u** exceeds the electron thermal velocity v_{Te}. In this article we investigate the flare phase that precedes the pre-heating phase. This is the situation when a sub-Dreicer electric field exists in the region under study. The expression for the instability growth rate was first obtained by Kryshtal (1997). Here we apply its modified version. For high-frequency longitudinal waves, instead of the standard ion–electron collision frequency ν_{ei} (see Equation (4)), we used the effective frequency ν_{ei}^* of ion–electron collisions (see *e.g.* Alexandrov, Bogdankevich, and Rukhadze, 1984):

$$\nu_{\mathrm{ei}}^* = \frac{1}{6\pi \varepsilon_0^2} \frac{Z e^4}{T_{\mathrm{e}}^{3/2}} \frac{\ln \Lambda n}{\sqrt{2\pi} m_e}, \tag{6}$$

where ε_0 is the dielectric permeability of the vacuum and Z is the atomic number. If the plasma frequency ω_{pe} and the electron-cyclotron frequency Ω_{e} are of the same order, *i.e.*

$$\omega_* \equiv \frac{\omega_{\mathrm{pe}}}{\Omega_{\mathrm{e}}} = O(1), \tag{7}$$

it makes sense to use ν_{ei}^* instead of ν_{ei}. It is easy to verify that Equation (7) is satisfied for a kilogauss magnetic field strength as well as for plasma density and temperature values from the FAL model (see *e.g.* Fontenla, Avrett, and Loeser, 1993) of the solar atmosphere. These parameters are realistic enough for a magnetic loop footpoint in the interval of heights $600 \leq H \leq 1400$ km. We assume that in this interval the condition $\nu_{\mathrm{ei}}, \nu_{\mathrm{ei}}^* \gg \nu_{0\mathrm{e}}, \nu_{0\mathrm{i}}$ is satisfied more strongly for the FAL model than for the MAVN (Machado *et al.*, 1980) and VAL (Vernazza, Avrett, and Loeser, 1981) models, which have been studied previously (Kryshtal, 1997, 1998). Here $\nu_{0\mathrm{e}}$ and $\nu_{0\mathrm{i}}$ are the frequencies of collisions of neutral atoms with electrons and ions, respectively. Additionally, we assume that the inequality

$$|k_*| \equiv \left| \frac{k_{\mathrm{z}}}{k_\perp} \right| \ll 1 \tag{8}$$

is satisfied for the components k_{z} and $k_\perp$ of the perturbation wave-vector, such that $|\mathbf{k}| = \sqrt{k_\perp^2 + k_z^2}$. The negative values of k_* correspond to the perturbation wave-vector that has a component $k_{\mathrm{z}} < 0$, *i.e.* is directed to the photosphere in our coordinate frame. The quasi-Bernstein modes discussed here are similar to the well-known neutralised Bernstein ion modes (Chen, 1984). In our case, $|k_*|$ does not have such a limit because the main frequency $\Omega_{\mathrm{e}} \gg \Omega_{\mathrm{i}}$ and the balance of charge separation is not rigid. But Equation (8) is very important to balance the Landau damping. We add one more condition

$$\Gamma \equiv \frac{\gamma}{\Omega_{\mathrm{e}}} \ll 1,$$

where γ is the instability growth rate. Through this condition, the process of instability development can be identified as an actual wave process. In the investigated region, we assume that the chromospheric pre-flare plasma is a low-β plasma (see *e.g.* Chen, 1984; Krall and Trivelpiece, 1973), *i.e.*

$$\beta \equiv \frac{8\pi n k_b T}{B_0^2} \ll 1, \tag{9}$$

where k_B is the Boltzmann constant and $T_e = T_i = T$. Additionally, we assume that

$$\beta \varepsilon_R \ll \sqrt{\frac{m_e}{m_p}} z_e |k_*|. \tag{10}$$

This means that the magnetic field $\mathbf{B}_0$ is considered as uniform for the relatively close connected pairs of plasma temperature and density. In Equation (10) we have used the notation

$$z_e \equiv \frac{k_\perp^2 v_{Te}^2}{\Omega_e^2} \tag{11}$$

for the kinetic electron plasma parameter (Krall and Trivelpiece, 1973). Therefore, the expression for the reduced growth rate Γ for the first harmonic can be written as

$$\Gamma \equiv \left.\frac{\gamma}{\Omega_e}\right|_{\omega=\omega_1} = \frac{C_1 + C_2}{D_1 + D_2}, \tag{12}$$

where

$$C_1 = \sqrt{\frac{\pi}{2}} \frac{(1-k_*^2)}{z_e} \exp(-z_e) \left[\frac{\omega_*^2}{k_*}\sqrt{\frac{1+\omega_*^2}{z_e}} - \frac{\varepsilon_R \sigma v^2 \omega_*^4}{8 z_e^3 k_*^6}\exp(-z_e) \right] S,$$

$$C_2 = -\frac{1}{2}\left[k_* v \varepsilon_R \frac{\sqrt{z_e}}{\omega_*^2} Q_1 - 2\sigma v Q_2 \right],$$

$$D_1 = Q_3 - 32 k_* \sigma v^2 \frac{\varepsilon_R}{\sqrt{z_e}\omega_*^{10}} Q_4,$$

$$D_2 = \sqrt{\frac{\pi}{2}} \frac{(1-k_*^2)}{z_e^{7/2}} \frac{\sqrt{1+\omega_*^2}}{k_*^2} S\omega_*^2 \exp(-z_e)\left[\frac{v\varepsilon_R}{k_*\sqrt{z_e}} - \frac{\sigma v}{4}\omega_*^4\exp(-z_e) \right].$$

Here we have used the notations

$$S \equiv \frac{z_e}{2}\exp\left[-\frac{\tau\omega_*^2}{4z_e^2 k_*^2}\right], \qquad \tau \equiv \frac{z_e}{4}\omega_*^2\exp(-z_e),$$

$$Q_1 \equiv 4 + 8z_e + (3-z_e)\omega_*^2 - \frac{4z_e}{\omega_*^4}\left(7\sigma^2 v^2 + 18k_*^2\right),$$

$$Q_2 \equiv 1 + \frac{z_e}{2} - \frac{\omega_*^2}{2} + 2z_e\frac{\sigma^2 v^2}{\omega_*^4} - 2z_e\frac{k_*^2}{\omega_*^4}\left(1 + \frac{\omega_*^2}{z_e}\right),$$

$$Q_3 \equiv 1 + z_e - \frac{\omega_*^2}{2} + 12z_e\frac{\sigma^2 v^2}{\omega_*^4} + 4z_e k_*^2\omega_*^6(1+2z_e)\left[3(1+2z_e) + 4\omega_*^2\right],$$

$$Q_4 \equiv 2 + z_e(8+7z_e) - (3+14z_e)\omega_*^2 - 20z_e\frac{\sigma^2 v^2}{\omega_*^4},$$

$$v = \frac{v_{ei}^*}{\Omega_e}.$$

To derive γ in Equation (12), *i.e.* to calculate the resonant term in the series expansion of the dispersion relation for pure electron Bernstein modes (Krall and Trivelpiece, 1973; Chen, 1984), we apply the method described in Kryshtal (1997) and Kryshtal, Gerasimenko, and Voitsekhovska (2012). This approach allows us to take into account the small corrections that include $\nu_{ei}^* \neq 0$, $\varepsilon_R \neq 0$ and $k_* \neq 0$ under the conditions given in Equations (5), (7), and (9). Obtaining a normalised growth rate for the first harmonic is difficult and tedious, even if the expression for γ is known for the second harmonic (Kryshtal, Gerasimenko, and Voitsekhovska, 2012). The calculations for each harmonic are independent and, therefore, have to be made separately. The free parameter σ takes into account the contribution of mutual collisions of all charged particles into the Bhatnagar–Gross–Krook (BGK) approximation of the collision integral (Alexandrov, Bogdankevich, and Rukhadze, 1984). The physical meaning of this integral and its use in calculations were described in detail before by Kryshtal and Kucherenko (1995, 1996) and Kryshtal, Gerasimenko, and Voitsekhovska (2012). The dispersion relation for pure electron Bernstein harmonics (with $k_z = 0$, $\nu = 0$ and $\varepsilon_R = 0$) has the form (Krall and Trivelpiece, 1973)

$$\omega^2 = m^2 \Omega_e^2 (1 + \alpha_m), \tag{13}$$

where $m = 1, 2, 3, \ldots$ is the harmonic number,

$$\alpha_m \equiv 2\omega_*^2 \frac{1}{z_e} I_m(z_e) \exp(-z_e),$$

and $I_m(z_e)$ is the associated Legendre polynomial. For $\varepsilon_R \neq 0$ and $k_z \neq 0$ the frequency ω in Equation (13) has to be replaced by ω', where

$$\omega' \equiv \omega - k_z u_e \equiv \omega - k_z \varepsilon_R \nu_{Te}. \tag{14}$$

In the long-wave approximation ($z_e \ll 1$) and under the conditions given in Equations (5), (7), and (9), the corrections to the dispersion relation, which appear as a result of taking into account the terms with $\varepsilon_R \neq 0$, $k_z \neq 0$, and $\nu \neq 0$, are small but still important for the growth rate value. Therefore, the dispersion relation for the first oblique quasi-Bernstein harmonics has the following form:

$$\omega_1^2 \approx (\omega')^2 = \Omega_e^2 \left[1 + 2\omega_*^2 \exp(-z_e) \right]. \tag{15}$$

The frequency ω_1 is very close to the upper-hybrid frequency, *i.e.*

$$\omega_{UHW}^2 = \omega_{pe}^2 + \Omega_e^2, \tag{16}$$

and coincides with it at $z_e \to 0$ and $\omega_* \ll 1$.

4. Results of the Calculations and Discussion

We derived the analytical expression for the reduced growth rate Γ of the first harmonic of pure electron weakly oblique Bernstein modes. The possible values of Γ were obtained for pre-flare chromospheric plasma and perturbation parameters near the footpoint of a flare loop, *e.g.* $1 \leq \sigma \leq 6$, $0.01 \leq z_e \leq 0.25$, $0.001 \leq |k_*| \leq 0.2$ and $5 \times 10^{-5} \leq \varepsilon_R \leq 1.5 \times 10^{-2}$. The calculated Γ sign change indicates waves with small constant amplitude, *i.e.* oscillations at the level of the thermal noise. The boundary values of the perturbation parameters for five possible modifications of the FAL solar atmosphere model (see Table 1) are summarised in Table 2 for the first and in Table 3 for the second oblique Bernstein harmonics. These modifications include two physical scenarios A and P. A corresponds to the case

Table 1 Main physical characteristics of the plasma from the FAL model of solar atmosphere. P1, P2, P3, A1, and A3 denote the five different sets of plasma parameters.

Modification	n_e $[10^{11} \text{ cm}^{-3}]$	$T_e = T_i$ [K]	B_0 [mT]	H [km]	ω_{pe} $[10^{10} \text{ s}^{-1}]$	Ω_e $[10^{10} \text{ s}^{-1}]$	ν^*_{ei} $[10^7 \text{ s}^{-1}]$
P1	2.50	5220	321	650	2.81	5.64	2.56
P2	3.02	5480	321	700	3.10	5.64	2.89
P3	4.79	7420	145	1380	3.80	2.56	1.23
A1	0.714	4390	321	650	1.51	5.64	0.95
A3	0.31	6150	145	1375	0.97	2.56	0.093

Table 2 The calculated values of the principal characteristics of the first oblique Bernstein harmonic instability for plasma parameters provided in Table 1.

Modification	$(\varepsilon_R)_{bound} \times 10^{-4}$	$(k_*)_{bound} \times 10^{-3}$	$(z_e)_{bound}$	$\Gamma_{(+)min}$ or $\Gamma_{(-)max}$
P1	–	–	–	-0.002
P2	–	–	–	-0.0023
P3	1.1	25.1	0.2	2.21×10^{-9}
A1	–	–	0.17	-8.8×10^{-7}
A3	1.0	155.4	0.15	1.31×10^{-4}

Table 3 The calculated values of the principal characteristics of the second oblique Bernstein harmonic instability for plasma parameters provided in Table 1.

Modification	σ	$(\varepsilon_R^{\|\|})_{bound} \times 10^{-4}$	$(k_*^{\|\|})_{bound} \times 10^{-3}$	$(z_e^{\|\|})_{bound}$	$\Gamma_{1+}^{\|\|}$
P1	–	10^{-2}	15.6	0.02	1.5×10^{-12}
P2	–	0.95	1.3	0.022	1.1×10^{-6}
P3	5	1.1	1.2	0.021	1.4×10^{-9}
A1	–	–	–	–	–
A3	5	1.05	17.4	0.019	2.8×10^{-11}

when at the specific height H the chromospheric temperature and plasma density are lowest, and P corresponds to the highest values of these parameters at the same height H (*see e.g.* Fontenla, Avrett, and Loeser, 1993). We note that the condition when the reduced growth rate Γ is lower than zero but extremely close to it corresponds to plasma processes with a large number of subcritical instabilities. To obtain the values of Γ we applied two additional conditions for z_e, k_* and ε_R. Specifically, (1) first positive value of Γ after Γ changes the sign from negative to positive has to be as low as possible, (2) if condition (1) is satisfied, the value of at least one of z_e, k_*, or ε_R should be at minimum. For the calculation, we applied condition (2) to z_e. As a result of this procedure, we identify a single set of values wherein the limit values of $(\varepsilon_R)_{bound}$, $(k_*)_{bound}$ and $(z_e)_{bound}$ are minimised. Formally, the condition $\Gamma > 0$ leads to small or very small changes in the limiting values of other parameters. However, when $z > (z_e)_{bound}$ the instability is present, but in this case, conditions (1) and (2) are violated. We note that for all cases the limiting values are defined by taking the first values at which the negative reduced growth rate Γ becomes positive.

Figure 1 The reduced growth rate of the instability, Γ as a function of k_* for FAL P3 model of the solar atmosphere. The kinetic electron plasma parameter $(z_e)_{bound} = 0.2$, $\sigma = 5$, and reduced collisional frequency $\nu = 4.77 \times 10^{-4}$. In this case, $(\varepsilon_R)_{bound} = 10^{-4}$ and $(k_*)_{bound} = -0.0251$.
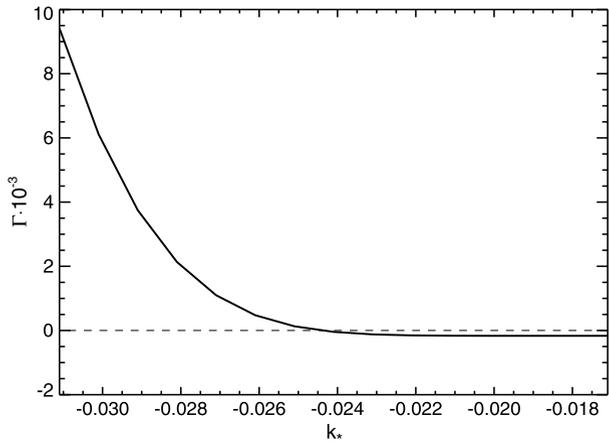
Figures 1 and 2 show the behaviour of $\Gamma \equiv \Gamma(z_e, k_*)$ at $z_e = 0.2$ for two modifications of the solar atmospheric model, *i.e.* FAL P3 and FAL A3, respectively. The curve $\Gamma(0.2, k_*)$ for FAL A3 corresponds to the subcritical instability. These instabilities are not marginal with respect to the parameters z_e and k_* because they do not exist at the limit of the corresponding interval of variation of the parameters. The odd powers of k_* in the expression for the reduced growth rate Γ (see Equation (12)) lead to an interesting result: the process of the initial appearance and subsequent development of the instability is asymmetric with respect to the sign change of k_*. Therefore, the behaviour of $\Gamma \equiv \Gamma(z_e, k_*)$ is very different for positive and negative values of k_* for the same and different solar atmospheric models. Figures 1 and 2 reflect this fact for FAL P3 and FAL A3 models, respectively.

The criterion for considering Landau damping to be weak has been proposed by Zhelezniakov, Zlotnik, and Ya (1975). This criterion was originally formulated as

$$\frac{|\omega - l\Omega_e|}{\sqrt{2}} \geq k_z V_{Te}.$$

We rewrite it as

$$k_* \ll \frac{|\sqrt{1 + 2\omega_*^2 \exp(-z)} - 1|}{\sqrt{2z_e}}. \qquad (17)$$

In this equation, $l = 1$ is the harmonic number. It is obvious that for FAL P3, $k_* \ll 1.81$ and the condition in Equation (17) is weaker than in Equation (8). For FAL A3, we obtain $k_* \ll 0.177$ and Equation (17) is stricter and more informative than Equation (8). For the chosen pre-flare plasma parameters (see Tables 1 and 2) the frequency of kinetic Alfvén waves (ω_{KAW}) in the three-wave process, UHW $\rightarrow$ KAW + EMW, is much lower than the frequency of the upper-hybrid pump wave (ω_{UHW}). Therefore, the frequency of electro-magnetic waves (ω_{EMW}) can be assumed to be approximately equal to the frequency of the first harmonic of pure electron weakly oblique Bernstein modes, *i.e.* $\omega_{EMW} \approx \omega_1$ (see Equation (15) for ω_1). We note that the threshold values of ε_R for the first and second harmonics are practically equal for FAL P3 and FAL A3 models (Kryshtal, Gerasimenko, and Voitsekhovska, 2012). Therefore, they can be simultaneously generated. The calculated frequencies of generated undamped waves are the following: $f_{theor}^{(1)}$(FAL P3) $\approx \Omega_e^{(1)}/2\pi \sqrt{1 + 2\omega_*^2 \exp(-z)} = 8.72$ GHz and $f_{theor}^{(1)}$(FAL A3) $= 4.52$ GHz. The superscripts (1) and (2) later in the text denote the first and second harmonics, correspondingly. These values are agree well with some observations in pre-flare region, for

Figure 2 The reduced growth rate of the instability, Γ, as a function of k_* for the FAL A3 model of the solar atmosphere. The kinetic electron plasma parameter is $(z_e)_{bound} = 0.2$, $\sigma = 5$ and the reduced collisional frequency is $\nu = 0.37 \cdot 10^{-4}$. In this case, $(\varepsilon_R)_{bound} = 10^{-4}$ and $(k_*)_{bound} = 0.1554$.
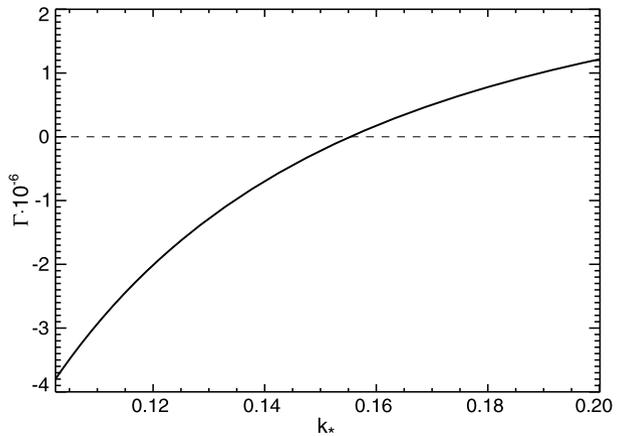
example in (i) the flare in NOAA AR 9077 (13 July 2000) (Bogod and Yasnov, 2009) with $f_{obs}^{(1)} \cong 5.0$ GHz (main peak), (ii) the flare in NOAA AR 10652 (25 July 2004) (Zharkova *et al.*, 2011) with $f_{obs}^{(1)} \cong 4.99$ GHz (burst-like structure in microwave emission before the initial phase of the flare), (iii) the flare in AR 11081 (N22W45, 12 June 2010) (Kashapova, Meshalkina, and Kisil, 2012) with $f_{obs*}^{(1)} \cong 4.6$ GHz, $f_{obs**}^{(1)} \cong 6.4$ GHz (main peak), $f_{obs***}^{(1)} \cong 9.4$ GHz (burst-like structures in microwave emission before the initial phase of the flare). In these examples the authors considered a model that includes energetic electron beams. In our analysis we investigated the non-beam model of the instability development. For FAL P1 the first harmonic is absent, but the second harmonic can appear. As a result of wave coalescence (see Equation (1)), the frequency of EMW can reach a value of $f_{theor}^{(2)}(\text{FAL P1}) = 4\Omega_e^{(1)}/2\pi \sqrt{1 + 2\omega_*^2 \exp(-z)} = 35.92$ GHz.

We also found that for FAL P1 model the instability of the first harmonic is absent and the second harmonic has an extremely low threshold value $(\varepsilon_R)_{bound} \approx 10^{-6}$. Therefore, the instability of the second harmonic can develop well before the first harmonic is generated (see also Kryshtal, Gerasimenko, and Voitsekhovska, 2012), and the frequency of the EMW has to be approximately the same as twice the frequency of the second pure electron harmonics.

The possibility of simultaneous generation of the first and second harmonics for FAL P3 and FAL A3 models indicates that harmonics with non-integer number (Willes and Robinson, 1996) can be present at the footpoint of the loop. The threshold values of ε_R for these harmonics are extremely low. Similar values, *i.e.* $10^{-4} \leq \varepsilon_R \leq 10^{-3}$, were obtained before by Kryshtal and Gerasimenko (2004, 2005) for kinetic Alfvén and kinetic ion-acoustic waves. The threshold values of ε_R for longitudinal waves are significantly higher, *i.e.* $0.01 \leq \varepsilon_R \leq 0.1$ (Kryshtal *et al.*, 2013; Kryshtal, Gerasimenko, and Voitsekhovska, 2014). We note that the second harmonic of oblique Bernstein waves for the FAL P1 model (Kryshtal, Gerasimenko, and Voitsekhovska, 2012) has the lowest threshold value of $\varepsilon_R = 10^{-6}$. The specific properties of the pre-flare plasma and wave perturbation in that region mean that the dispersion relations for the upper-hybrid wave and first harmonics of pure electron weakly oblique Bernstein modes might coincide. The high-quality factors of the investigated instabilities show that their development can be considered as a periodic wave process that corresponds to the generation of small amplitude (*e.g.* at the level of thermal noise) undamped waves.

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