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Solitary wave propagation in solar flux tubes

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The aim of the present work is to investigate the excitation, time-dependent dynamic evolution, and interaction of nonlinear propagating (i.e., solitary) waves on vertical cylindrical magnetic flux tubes in compressible solar atmospheric plasma. The axisymmetric flux tube has a field strength of 1000 G at its footpoint, which is typical for photospheric regions. Nonlinear waves that develop into solitary waves are excited by a footpoint driver. The propagation of the nonlinear signal is investigated by solving numerically a set of fully nonlinear 2.0D magnetohydrodynamic (MHD) equations in cylindrical coordinates. For the initial conditions, axisymmetric solutions of the linear dispersion relation for wave modes in a magnetic flux tube are applied. In the present case, we focus on the sausage mode only. The dispersion relation is solved numerically for a range of plasma parameters. The equilibrium state is perturbed by a Gaussian at the flux tube footpoint. Two solitary solutions are found by solving the full nonlinear MHD equations. First, the nonlinear wave propagation with external sound speed is investigated. Next, the solitary wave propagating close to the tube speed, also found in the numerical solution, is studied. In contrast to previous analytical and numerical works, here no approximations were made to find the solitary solutions. A natural application of the present study may be spicule formation in the low chromosphere. Future possible improvements in modeling and the relevance of the photospheric chromospheric transition region coupling by spicules is suggested. © 2006 American Institute of Physics.

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I. INTRODUCTION

Observational data clearly show that the solar atmosphere is heavily structured. Recent magnetohydrodynamic (MHD) wave studies in magnetic structures (e.g., flux tubes in the solar atmosphere) have given a boost by TRACE and SOHO observations since the MHD waves became directly observable by these satellites (see the reviews in Ref. 1). Observations show that the MHD waves are mainly linear by the time they appear at coronal temperatures. On the other hand, there is little progress made on how these waves propagate to coronal heights; where do they come from; what is their connection to the transition region (TR) or to even lower parts of the atmosphere; or is there a connection at all? Very recent observations of the transition region, in particular spicules and moss oscillations, detected by TRACE and by SUMER on board SOHO, may bring us closer to the origin of the running (propagating) waves of coronal loops. The correlations on arcsecond scales between chromospheric and transition region emission in active regions were studied in Ref. 2. The discovery of active region moss, i.e., dynamic and bright upper transition region emission at transition region heights above active region (AR) plage, provides a powerful diagnostic tool to probe the structure, dynamics, energetics, and coupling of the magnetized solar chromosphere and transition region. In Ref. 2, the interaction of the chromosphere with the upper TR, by studying correlations (or the lack thereof) between emission at varying temperatures, was carried out in great detail: from the low chromosphere (Ca II K-line), to the middle and upper chromosphere (Hα), to the low transition region (C IV 1550 Å at 0.1 MK), and the upper transition region (Fe IX/X 171 Å at 1 MK and Fe XII 195 Å at 1.5 MK). The high cadence (24–42 s) data sets obtained with the Swedish Vacuum Solar Telescope (SVST, La Palma) and TRACE allowed us to find a relation between upper transition region oscillations and low-lying photospheric oscillations. The correlation analysis gave some partial answers to the question of how the heating mechanisms of the chromosphere are related and whether the spatial and temporal variability of moss (and spicules) can be used as diagnostics for coronal heating. Next, in Ref. 3, the intensity oscillations in the upper TR above AR plage were analyzed. A possible role of a photospheric driver in the appearance of moss (and spicule) oscillations was suggested. Wavelet analysis of the observations (by TRACE) verifies strong (~5–15%) intensity oscillations in the upper TR footpoints of hot coronal loops. A range of periods from 200 to 600 s, typically persisting for four to seven cycles, was found. A preliminary comparison of photospheric vertical velocities (using the Michelson Doppler Imager on board SOHO) revealed that some upper TR oscillations show a correlation with solar global acoustic p modes in the photosphere. In addition, the majority of the upper TR oscillations are directly associated with upper chromospheric oscillations observed in Hα, i.e., periodic flows in spicular structures. The presence of such strong oscillations at low heights (of order 3000 km) provides an ideal opportunity to study the propagation of oscillations from photosphere and chromosphere into the TR and corona (see,

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for example, Ref. 4). It can also help us to understand the magnetic connectivity in the chromosphere and TR, and shed light on the source of chromospheric mass flows such as spicules. Especially this latter aspect gave us the motivation to study how a signal (linear or weakly nonlinear) excited (periodically or solitary) at the photospheric level would be manifest higher up in a stratified magnetic solar atmosphere (see also Ref. 5).

In the present work, we report on some initial results obtained by studying the propagation of linear and nonlinear waves (waves of the solitary type) that are excited by a footpoint driver. The propagation of the signal is investigated by solving numerically a set of fully nonlinear 2D MHD equations. First, we investigate the time-dependent evolution of the fast and slow sausage surface waves in a magnetic flux tube. Next, we compare our results with asymptotic analytical solutions as given in Ref. 6 that were obtained by using the method of multiple scale expansion. The nonlinear stage of the present numerical simulations is compared with simulations carried out in Ref. 7, where only the approximate Leibovich-Roberts (LR) evolution equation

\[ \frac{df}{dt} + c_r \frac{df}{dz} + \beta \frac{df}{dz} + \alpha \int_{-\infty}^{+\infty} \frac{f(s,t)}{[\lambda^2 + (z-s)^2]^{1/2}} ds = 0 \]

was solved numerically as opposed to the current full MHD solution. Here f denotes the velocity perturbation along the flux tube. The parameters \( \alpha, \beta, \) etc. are given in Ref. 8, where this equation was first analytically derived. We also show that a solution similar to the asymptotic solution of the LR equation found in Ref. 6 in a more general case indeed can be found in the framework of a full MHD description.

II. THE BASIC EQUATIONS AND ASSUMPTIONS

The analysis of the linear or nonlinear excitation and wave propagation in a magnetic flux tube is of fundamental importance. Pioneering analytical investigations of the linear problem in slab and tube geometry were carried out by, e.g., Refs. 9–14. The study of nonlinear waves in flux tubes was investigated by, e.g., Refs. 8 and 15–20 and many others. The main issue is not just to derive the governing equation for weakly nonlinear perturbations, which has been more or less successful, but to find solutions to this governing equation. The latter, one may say, was less successful unless approximation theories (long wavelength or thin flux tube; method of multiple scale expansion) were used. Alternatively, the analytically derived approximation governing equations were solved by numerical schemes. However, to the best of our knowledge, no attempt was made to solve, even numerically, the full nonlinear MHD equations in the nonlinear regime for a vertical homogeneous flux tube.

In the present work we consider, numerically, the wave propagation in a cylindrical magnetic flux tube in a nonstratified solar atmosphere. The set of the full MHD equations reads as follows:

\[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{\nabla p}{\rho} + \frac{1}{\mu \rho} (\nabla \times \mathbf{B}) \times \mathbf{B}, \]

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \]

\[ \nabla \cdot \mathbf{B} = 0, \]

\[ p = p_0 \left( \frac{\rho}{\rho_0} \right)^\gamma, \]

where \( \rho \) is the density, \( p \) the pressure, \( \mathbf{V} \) the velocity, \( \mathbf{B} \) magnetic induction, and \( \gamma = 5/3 \) is the adiabatic index. These equations are solved in cylindrical coordinate system \( r, \phi, z \). We denote the tube radius by \( r_0 \), and all dependent variables inside the tube have no index, while outside the tube they are denoted with index \( e \). We are interested in waves that are much longer than the lateral dimensions of the magnetic field. The coordinate system describing the equilibrium model is shown in Fig. 1. Because we assume cylindrical symmetry, the azimuthal components of the velocity and magnetic field are set equal to zero. \( \rho_0 \) and \( p_0 \) are the undisturbed density and the pressure inside the flux tube; \( \rho_{e0} \) and \( p_{e0} \) are the undisturbed density and the pressure outside the flux tube; there is no undisturbed velocity, i.e., \( V_0 = 0 \). Neglecting gravity, the static state of equilibrium is defined by

\[ p_0 + B_0^2 / 2\mu = p_{e0}, \]

where \( \mathbf{B}_0 = (0,0,B_0) \) is the undisturbed magnetic field inside the flux tube. Let us now consider small velocity perturbations \( \mathbf{V} = [u(r,0,z),0,w(r,0,z)] \), pressure \( p \), density \( \rho \), and magnetic field \( \mathbf{b} = [b_r(r,0,z),0,b_z(r,0,z)] \) inside and outside the magnetic tube. The dynamics of wave modes may be described by the well-known equation as given in Ref. 21.
\[ \frac{\partial^2}{\partial t^2} \left( \frac{\partial^2}{\partial r^2} - (V^2 + C_0^2) \right) + V^2 A C_0^2 \frac{\partial^2}{\partial r^2} V^2 \Delta = 0, \]  

where \( \Delta = \nabla \cdot \mathbf{u} \). Assuming that \( \Delta = R(r) \omega^{i(\omega t - n \theta - k \varphi)} \) and substitution into (7) yields

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial R}{\partial r} \right) + \left( k_0^2 + \frac{n^2}{r^2} \right) R = 0, \]  

as obtained in Ref. 14, where

\[ k_0^2 = \frac{(\omega^2 - k^2 V_A^2)(\omega^2 - k^2 C_0^2)}{(V_A^2 + C_0^2)(\omega^2 - k^2 C_e^2)}. \]  

Here \( V_A = B_0 \sqrt{\mu_0 \rho_0} \) is the Alfvén speed, \( C_0 = (\gamma \rho_0 / \rho_0)^{1/2} \) is the sound speed inside the tube, and \( C_e = C_0 V_A / \sqrt{(C_0^2 + V_A^2)} \) denotes the tube speed. The state outside of the tube is described by a similar equation (all dependent variables with index \( e \)). In the present work, we consider the outside region to be nonmagnetic. Therefore,

\[ k_e = \frac{\omega^2 - k^2 C_e^2}{C_e^2}, \]  

where \( C_e = (\gamma \rho_0 / \rho_0)^{1/2} \) is the sound speed outside the tube. Equation (8) corresponds to Bessel’s equation with variable \( k_0^2 \). The general solution of (8) has the form, provided \( k_e^2 > 0, \)

\[ R(r) = \begin{cases} A_{ne} I_n(k_0 r) + B_{ne} Y_n(k_0 r), & r < r_0, \\ A_{ne} I_n(k_e r) + B_{ne} Y_n(k_e r), & r > r_0, \end{cases} \]  

where \( I_n \) and \( Y_n \) are the Bessel functions of the first and second kind, respectively. These solutions correspond to spatially oscillating waves. However, for \( k_e^2 = -m_0^2 < 0 \) and \( k_e^2 = -m_e^2 < 0 \) we obtain solutions describing surface waves,

\[ R(r) = \begin{cases} C_{00} I_0(m_0 r) + D_{00} K_0(m_0 r), & r < r_0, \\ C_{0e} I_0(m_e r) + D_{0e} K_0(m_e r), & r > r_0, \end{cases} \]  

where \( I_n \) and \( K_n \) are the modified Bessel functions of the first and second kind. The amplitude of the perturbations at the origin and at infinity \( (r \to \infty) \) must be finite, so the constants \( D_{00}, B_{00}, \) and \( C_{0e} \) are equal to zero. By substituting these solutions back into Eqs. (1)–(5) for surface waves only, we obtain, for the perturbations inside the flux tube (see, e.g., Ref. 13),

\[ \rho = iC_{00} \rho_0 C_0^2 \omega^{-i} I_0(m_0 r), \]

\[ u = C_{00} \omega^2 - k^2 C_0^2 \frac{d}{dr} I_0(m_0 r), \]

\[ w = - C_{00} \frac{C_0^2}{\omega^2} ik I_0(m_0 r), \]

\[ b_r = - \frac{k}{\omega} B_0, \]

\[ b_\varphi = iC_{0e} \omega^2 - k^2 C_e^2 B_0 I_0(m_e r), \]  

\[ p = iC_{0e} \rho_0 C_e^2 \omega^{-i} I_0(m_e r). \]  

Similar expressions hold for the external region,

\[ \rho_e = iD_{0e} \rho_0 - iK_0(m_e r), \]

\[ u_e = D_{0e} \omega^2 - k^2 C_e^2 \frac{d}{dr} K_0(m_e r), \]

\[ w_e = - D_{0e} \frac{C_e^2}{\omega^2} ik K_0(m_e r), \]

\[ b_r = 0, \]

\[ b_\varphi = 0, \]

\[ p_e = iD_{0e} \rho_0 C_e^2 \omega^{-i} K_0(m_e r). \]  

The conditions for matching the inside and outside solutions at \( r = r_0 \) are

\[ u_e(r_0) = u(r_0), \]

\[ p_e = p + \frac{1}{\mu} B_0 b_\varphi. \]  

Eigenfunctions of the surface mode, in a thin flux tube, are shown in Fig. 2. The amplitude of the radial velocity inside the tube increases linearly, while outside the tube it exponentially decreases [Fig. 2(a)]. The full pressure is constant across the tube [see Fig. 2(c)]. The longitudinal velocity at the boundary of the tube has discontinuity [Fig. 2(d)]. Eliminating the velocity amplitudes and applying the boundary conditions yields the general dispersion equations; see, e.g., Refs. 9–11, 13, and 14. In the case of body waves that are spatially oscillating inside the waveguide but evanescent outside.
and then determine those values of the phase speed 

cause case of body waves, of course, we find many solutions be-

Ref. 14 has the approximate solution for the slow surface sausage 

FIG. 4. (Color online) The phase speed (km/s) of modes under photospheric conditions \( V_A > C_0 > C_T \). Many slow body waves are shown.

modes side \( (m^2 > 0, m^2 > 0) \), the cylindrically symmetric mode (i.e., \( n=0 \), sausage mode) is the solution of

In the case of surface waves \( (m^2 > 0, m^2 > 0) \) the dispersion relation for the cylindrically symmetric mode is as follows:

Numerical solutions of the dispersion relations for surface and body waves are shown in Figs. 3 and 4 (see also Ref. 14). The numerical procedure is to select a value for \( k r_0 \), and then determine those values of the phase speed \( V_{ph} = \omega / k \) for which the dispersion relation is satisfied. In the case of body waves, of course, we find many solutions because \( J_n \) is oscillatory. For the numerical calculations, we applied typical photospheric conditions (see Table I). The typical diameter of an intense flux tube is \( \approx 100–300 \) km.

In the thin-flux-tube approximation \( (k r_0 \ll 1) \), Eq. (18) has the approximate solution for the slow surface sausage mode as given in Ref. 6.

\[ \omega = C_T k + 2 \beta k^3 \left( \ln \frac{\alpha |k|}{2} + 0.577 \right) + O(k^5 \ln |k|), \] (19)

where

\[ \beta = \frac{\rho_0 C_T^2}{8 \rho_0 V_A^2 r_0^2} \) \] \( \alpha^2 = \left( 1 - \frac{C_T^2}{C_s^2} \right) r_0^2. \]

Figure 5 shows the phase speed \( V_{ph} \) as a function of the dimensionless wave number \( (k r_0) \). We see that for \( k r_0 \ll 1 \), the exact numerical solution of (18) (solid line) coincides with the approximate solution for the long-wavelength limit (19) (dot-dashed line). The dashed line indicates the tube speed.

The boundary of the flux tube can be found by using the equation valid at \( r = r_0 + \eta(t, z) \) (see, e.g., Ref. 6)

\[ u = \frac{\partial \eta}{\partial t} + \nu \frac{\partial \eta}{\partial z}. \] (20)

III. NUMERICAL CALCULATIONS

To solve numerically the full MHD problem of the time-dependent evolution of nonlinear surface sausage waves on vertical cylindrical magnetic flux tubes, the Versatile Advection Code (VAC) is applied (see, e.g., Ref. 22). All calculations were carried out by using the combinations of the total variations diminishing (TVD) and the TVD Lax-Friedrich (TVDLF) methods. We apply the free boundary conditions at the domain boundaries in order to achieve reasonable reduc-

TABLE I. The typical photospheric conditions.

<table>
<thead>
<tr>
<th>( B_0 ) (G)</th>
<th>( V_A ) (m/s)</th>
<th>( C_0 ) (m/s)</th>
<th>( C_T ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>tube which is cooler than its surroundings</td>
<td>1000</td>
<td>( 9 \times 10^3 )</td>
<td>( \approx 6.4 \times 10^3 )</td>
</tr>
<tr>
<td>intense cool tube</td>
<td>1000</td>
<td>( 9 \times 10^3 )</td>
<td>4.5 ( \times 10^3 )</td>
</tr>
</tbody>
</table>

FIG. 5. The phase speed (m/s) of a slow surface mode for a tube that is cooler than its surroundings. The exactly (solid line) and approximately (dot-dashed line) solutions.

FIG. 3. (Color online) The phase speed (km/s) of modes under photospheric conditions \( V_A > C_0 > C_T \). Many slow body waves are shown.
tion of reflection. For the numerical calculations, the set of the full MHD equations, that are actually solved, reads as follows:

\[
\frac{\partial \rho V}{\partial t} + \nabla \cdot (\rho V V - B B) + \nabla p_{\text{tot}} = - (\nabla \cdot B) B, \tag{21}
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0, \tag{22}
\]

\[
\frac{\partial e}{\partial t} + \nabla \cdot (e - BB \cdot V + V p_{\text{tot}}) = - (\nabla \cdot B) B \cdot V, \tag{23}
\]

\[
\frac{\partial B}{\partial t} + \nabla \cdot (VB - BV) = (\nabla \cdot B) V, \tag{24}
\]

\[
p = (\gamma - 1)(e - \rho V^2/2 - B^2/2), \tag{25}
\]

\[
p_{\text{tot}} = p + B^2/2,
\]

with conservative variable density \(\rho\), momentum density \(\rho V\), total energy density \(e\), and magnetic field \(B\). The magnetic field is rescaled as \(B = B/\sqrt{\mu} \). The terms that are proportional to \(V \cdot B\) are necessary in order to eliminate numerical problems related to the divergence of the magnetic field. For more details on VAC, see, e.g., Ref. 22.

At the boundary of the flux tube, the full pressure balance (16) should be satisfied after starting the calculations. Numerically we studied two distinct cases: (i) linear and (ii) nonlinear stage of wave propagation along the tube. For the linear MHD wave simulations, (i) we use a cylindrically symmetric domain \([0, 80r_0]\) (400 grid points) in the \(z\) direction and \([0, 2r_0]\) (140 grid points) in the \(r\) direction. Initially, the vertical magnetic field is perturbed along the tube, see Fig. 6.

After starting the linear calculations, the boundary profile monotonically increases toward a \(\sin\) profile (see Figs. 7 and 8) for various snapshots. These profiles are symmetric about the \(z\) axis. Figure 8 shows the vertical perturbations for all variables. Observe that the full pressure is constant across the whole domain. It corresponds to the condition of pressure balance (16) achieved properly in the numerical experiment.

Next, in Fig. 9 we plot a snapshot of the radial velocity at the same time when Fig. 8 is shown. The perturbation profile in the \(r\) direction has the constant shape which corresponds perfectly to the analytical solutions (13), (14), and Fig. 2.

In what follows, we plot the numerical solutions of the full set of the MHD equations for the nonlinear case (ii). For the latter, we employ the symmetric domain \([0, 320r_0]\) (1600 grid points) in the \(z\) direction and \([0, 2r_0]\) (150 grid points) in the \(r\) direction. Initially, the magnetic field is perturbed at the footpoint. The perturbation has a Gaussian spatial distribution,

\[
b_z = B_0 \left(1 + C I_0(m_0 r_0) \right) e^{-\left(z^{2} - 16r_0^2\right)^2}, \tag{26}
\]

where \(C=0.15\).

After the initial perturbation, the evolution of any variables with time can be observed throughout the whole computational domain. As an example, we present results of the simulations for the \(u\) and \(w\) components of the velocity. We found actually two distinct solutions already predicted in the
asymptotic analytical studies: (a) a fast nonlinear sausage surface wave, which propagates with a velocity approximately equal to $C_e$, and (b) a slow nonlinear sausage surface wave, which propagates with $C_T$.

A. Nonlinear solution close to $C_e$

The full time necessary for the formation and propagation of a fully developed fast nonlinear sausage surface wave is, formally, divided into four stages. First, linear propagations are shown in Fig. 10. During this period of time, the initial perturbations propagate with small (linear) amplitudes. However, this period of time is rather short. The second stage is where dispersion dominates. At this stage, the major role on the propagation of the initial signal is the dispersive effect. This solution is nonlinear in the sense that the amplitude is increasing with the propagation speed. This behavior is typical for cnoidal waves; see, e.g., Ref. 23.

The third stage is nonlinear (as shown in Fig. 11). The wave amplitude of the oscillatory solution grows and steepens further, and is balanced by the tendency for dispersive effects to spread the wave. The dispersion prevents wave overturning and shock development. Finally, the fourth stage is solitary (Fig. 12). At this stage, we see the formation of

FIG. 9. (Color online) 2D snapshot of the radial velocity perturbation $u$ for the linear case (i). The velocity is measured in m/s. $x$ corresponds to the radial direction (in meters), and $z$ is the direction along the tube (in meters).

FIG. 11. The time-dependent evolution of the initial signal at the third stage. The vertical line denotes the footpoint.

FIG. 12. The time-dependent evolution of the initial signal at the last (the fourth) stage. Again, the vertical line here indicates the footpoint region.
solitary waves (nonlinear waves of the solitary type) which are propagating without any permanent change with constant speed of propagation (\( \sim C_e \)). This behavior is typical for solitary waves. Analytically the propagation of the nonlinear fast sausage surface waves in a magnetic slab embedded in a magnetic-free environment was considered by Ref. 24 in the limit of small amplitude.

In Fig. 13, we plot a snapshot of the radial velocity at the same time as that of Fig. 12. The perturbation profile in the \( r \) direction inside the tube is linear, as expected, and corresponds approximately to the Bessel function of the first kind. Outside the tube, the profile is proportional to the Bessel function of the second kind (this is a typical for behavior surface waves). In order to establish whether this wave is indeed a solitary wave, we have to analyze their interaction with each other. In Fig. 14, the temporal evolution of solitary wave interaction for type (a) is displayed. Two consecutive waves are launched. Since their initial amplitudes are different, they will catch up with each other. After their collision, the profiles show only very small deviations from the initial profiles. In the case presented here, the amplitude of the propagating signal is decreased by (\( \sim 5-10\% \)). This effect can be explained by radiation in the form of small oscillations on the tails of signals after the interaction (see, e.g., Ref. 23). Therefore, we may conclude that these waves are not solitary waves in the very strict sense of this word. However, these waves can be labeled as nonlinear waves of the solitary type. A reasonable check for this statement would be to plot the width, \( d \), of the wave as a function of its amplitude, \( A \). In Fig. 15, the solid line represents the dependence of solitary wave width \( d \) on amplitude of wave \( A \). We see that the numerical solution for the nonlinear wave which propagates with \( C_e \) speed is close to the Korteweg–de Vries solution.

### B. Nonlinear solution close to \( C_T \)

Next, we study slow nonlinear sausage surface waves. After a time of about \( t=180s \), the initial Gaussian profile changes. Sharp peaks develop in the front that cause profile asymmetry. These large gradients develop in the front as a result of the wave breaking effect. The amplitude of the profile is monotonically increasing (Fig. 16).

Perhaps the decay of this nonlinear wave will proceed until the profile reaches the critical amplitude. The critical amplitude for the numerical solution of the LR equation was found by Ref. 7 in an asymptotic way. The numerical solution found in the current case, in the form of a slow solitary wave, shows very similar properties to the analytical solution of the LR equation found by Ref. 6 in the general form.
linear signal propagating with tube speed. Where all parameters are explained in details in Ref. 6. sausage mode with the tube speed waves. MHD equations are obtained numerically in the form of two nonstratified magnetic flux tube. The solutions of the full propagation of an initial Gaussian perturbation in a straight mately equals the sound speed outside the flux tube. These waves travel with the speed which approxi-

waves that were analytically found for slab geometry using the method of matched asymptotic approximations. The width-amplitude dependence of these waves, however, shows a resemblance to those of KdV type solitary waves. This behavior is also speculated in Ref. 12. The interaction of the nonlinear waves with tube speed was more complex. Although the initial shapes were preserved after their collision, we also found minor amplitude decreases. It is very hard to estimate whether this energy loss is due to a nonlinear phase or due to numerical viscosity. Further detailed investigations of the propagation and interaction of these types of solitary waves are necessary in order to evaluate the slow nonlinear sausage surface wave formation.

IV. CONCLUSION

In the present paper, we study the linear and nonlinear propagation of an initial Gaussian perturbation in a straight nonstratified magnetic flux tube. The solutions of the full MHD equations are obtained numerically in the form of two sets of nonlinear waves. First, we found nonlinear surface waves (of solitary type) that develop and propagate in a flux tube. These waves travel with the speed which approximately equals the sound speed outside the flux tube (fast sausage mode). Next, we also found a wave that propagates with the tube speed (slow sausage mode). The behavior of this second solution to the full nonlinear MHD equations is similar (at their early stage of formation) to those shown by Ref. 7. Interactions of two solitary waves with external sound speed were also investigated. We found that the signals kept their identity after the interaction, however they have shown no phase shifts during their interaction (Fig. 17). This feature is typical for, e.g., Benjamin-Ono-type solitary waves.

\[
\frac{\partial f}{\partial t} + c_T \frac{\partial f}{\partial z} + b f \frac{\partial f}{\partial z} + \frac{B}{\alpha c_T} \frac{\partial^3 f}{\partial z^3} \int_{-\infty}^{z'} F[\alpha^{-1}(z-z')]f(z')dz' = 0,
\]

where all parameters are explained in details in Ref. 6.

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